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The Effect of Curvature on the Transport Coefficients of Thermal Diffusion in Concentric-Tube Columns

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Abstract

The effect of curvature on the transport coefficients of thermal diffusion in concentric-tube columns has been investigated and the results were obtained graphically. It has been found that the results for the nearly plane case obtained by Yeh and Ward still can be used for the extreme cylindrical case, with the transport coefficients multiplied by the modifying factors presented.

INTRODUCTION

The thermogravitational thermal diffusion column, introduced by Clusius and Dickel (*2, 3*), is an unusual device for the separation of liquid or gas mixtures. A complete presentation of the theory of the Clusius-Dickel column was made by Furry et al. (*4–6*). A more detailed study of the mechanism of separation in the Clusius-Dickel column indicates that proper control of the convective strength might lead to improved separation (*1, 8–13*).

The transport phenomenon of thermal diffusion in a concentric-tube column can be considered as that of a flat-plate one when the annular spacing is small compared with the tube diameters (*7*). However, the effect of curvature should be taken into consideration because the ratio of tube diameters is far from unity, and in this case the approximation of a nearly plane case will lead to serious error. The extension of the column theory to take account of the cylinder shape of the actual apparatus is simple in principle, but is much more complicated in execution.

The equation of separation for maxwellian gases only in a concentric-tube thermal diffusion column was obtained by Furry and Jones (5). It is the purpose of this work to develop the theory for any fluids in a concentric-tube thermal diffusion column and to find the modifying factors between the transport coefficients of the flat-plate and concentric-tube columns for convenient use.

EQUATION OF SEPARATION

The temperature gradient applied between the tube surfaces of a concentric-tube thermal diffusion column has two effects: (1) a flux of one component of the solution relative to the other is brought about by thermal diffusion, and (2) convective currents are produced parallel to the tube surface owing to density differences. The combined result of these two effects is to produce a concentration difference between the two ends of the column which is generally much greater than that obtained by the static method. Meanwhile, the concentration gradient produced by the combined effects of thermal diffusion and convection acts to oppose thermal diffusion and limits the separation. Figure 1 illustrates the flows and fluxes prevailing in a continuous-flow column.

Since the space between the tube surfaces of the column is generally small, we may assume that the convective flow produced by the density gradient is laminar and that the temperature distribution is determined by conduction in the r-direction only. We also assume that the convection velocity is in the z-direction only, that both end effects can be neglected, and that the mass fluxes due to thermal and ordinary diffusion are too small to affect the velocity and temperature profiles. Applying the appropriate equations of motion and energy gives the following steady-state velocity profiles.

In the enriching section:

$$v_z = \frac{\beta_T g R^2(\Delta T)^2}{4\mu}\left\{\left(A_1 + \frac{1}{\ln k}\right)(\xi^2 - 1) - \frac{1}{\ln k}\left[\frac{1}{\ln k}(k^2 - 1 - k^2 \ln k) + A_1(k^2 - 1) + \xi^2\right]\ln \xi\right\} \tag{1}$$

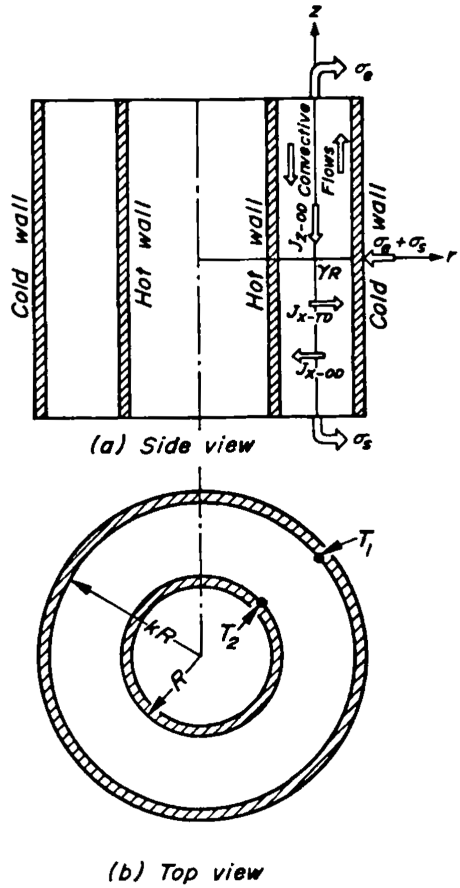

FIG. 1. Schematic diagram of a concentric tube thermal diffusion column.

where

$$A_1 = \frac{\bar{T} - T_1}{\Delta T}$$

$$= \frac{[(\frac{7}{4}k^4 - k^4 \ln k - k^2 - \frac{3}{4}) \ln k - (k^2 - 1)^2 + \mu\sigma_e/2\rho\beta_T\pi R^4(\Delta T)g]}{(k^2 - 1)[(k^2 - 1) - (k^2 + 1) \ln k] \ln k} \tag{2}$$

The above equations apply with σ_e replaced by $-\sigma_s$ in the stripping section.

The horizontal mass flux of component 1 of a binary mixture is related to the velocity by the differential mass balance equation

$$\frac{\partial}{\partial \xi}(\xi J_r) + \rho R \xi v_z \frac{\partial c}{\partial z} = 0 \tag{3}$$

In obtaining the above equation the assumption was made that diffusion in the z-direction is negligible. If the bulk flow in the r-direction is negligible, the expression for J_r in terms of the two contributions, ordinary and thermal diffusion, is

$$J_r = \frac{D\rho}{R}\left(-\frac{\partial c}{\partial \xi} + \frac{\alpha c\bar{c}}{T}\frac{dT}{d\xi}\right) \tag{4}$$

The concentration profile may be calculated by combining Eqs. (1), (3), and (4), and intergrating with the following conditions:

$$\text{at } \xi = 1 \qquad J_r = 0 \tag{5}$$

$$\text{at } \xi = k \qquad J_r = 0 \tag{6}$$

$$\text{at } \xi = \xi \qquad c = c(\xi, z) \tag{7}$$

$$\text{at } \xi = \gamma = \frac{k+1}{2} \qquad c = c_0(\gamma, z) \tag{8}$$

The solution is

$$\begin{aligned} c = c_0 &+ \frac{\alpha c\bar{c}}{T}\frac{(\Delta T)}{\ln k}\ln\frac{\xi}{\gamma} + \frac{\beta_T g R^4(\Delta T)}{4\mu D}\phi(z)\Bigg\{\left(A_1 + \frac{1}{\ln k}\right) \\ &\times \left[\left(\frac{\xi^4}{16} - \frac{\xi^2}{4} + \frac{1}{4}\ln\frac{\xi}{\gamma} - \frac{\gamma^4}{16} - \frac{\gamma^2}{4}\right) + A_2\left(\frac{\xi^5}{25} - \frac{\xi^3}{9} + \frac{2}{15}\ln\frac{\xi}{\gamma} - \frac{\gamma^5}{25} + \frac{\gamma^3}{9}\right)\right] \\ &- \frac{1}{\ln k}\Bigg[\left(\frac{\xi^4 \ln \xi}{16} - \frac{\xi^4}{32} + \frac{1}{16}\ln\frac{\xi}{\gamma} - \frac{\gamma^4 \ln \gamma}{16} - \frac{\gamma^4}{32}\right) \\ &+ A_2\left(\frac{\xi^5 \ln \xi}{25} - \frac{2\xi^5}{125} + \frac{1}{25}\ln\frac{\xi}{\gamma} - \frac{\gamma^5 \ln \gamma}{25} + \frac{2\gamma^5}{125}\right)\Bigg] \\ &- \frac{1}{\ln k}\left[\frac{1}{\ln k}(k^2 - 1 - k^2 \ln k) + A_1(k\gamma - 1)\right] \\ &\times \Bigg[\left(\frac{\xi^2 \ln \xi}{4} - \frac{\xi^2}{4} + \frac{1}{4}\ln\frac{\xi}{\gamma} - \frac{\gamma^2 \ln \gamma}{4} + \frac{\gamma^2}{4}\right) \\ &+ A_2\left(\frac{\xi^3 \ln \xi}{9} - \frac{2\xi^3}{27} - \frac{1}{9}\ln\frac{\xi}{\gamma} - \frac{\gamma^3 \ln \gamma}{9} - \frac{2\gamma^3}{27}\right)\Bigg]\Bigg\} \end{aligned} \tag{9}$$

In obtaining the above solution, it was assumed that the quantity $\alpha c\bar{c}/T$ appearing in the thermal diffusion term was independent of r, and $dT/d\xi = (\Delta T)/(\xi \ln k)$ as obtained from the energy equation was used, and it was assumed that $\partial c/\partial z = (1 + A_2\xi)\phi(z)$, where

$$A_2 = -[2\mu\sigma_e/\rho\beta_T\pi g R^4 k(\Delta T)] \Bigg/ \left\{\left(A_1 + \frac{1}{\ln k}\right)\left(\frac{k^4}{5} - \frac{k^2}{3} + \frac{2}{15k}\right) - \frac{1}{\ln k}\left(\frac{k^4 \ln k}{5} - \frac{k^4}{25} - \frac{1}{25k}\right) - \frac{1}{\ln k}\left[\frac{1}{\ln k}(k^2 - 1 - k^2 \ln k) + A_1(k^2 - 1)\right] \times \left(\frac{k^3 \ln k}{3} - \frac{k^2}{9} + \frac{1}{9k}\right)\right\} \tag{10}$$

The rate of mass transport of component 1 in the z-direction is given by

$$\tau_1 = \int_R^{kR} \rho c v_z(2\pi r)\, dr - \int_R^{kR} \rho D \frac{\partial c}{\partial z}(2\pi r)\, dr \tag{11}$$

Combining Eqs. (1), (9), and (11), replacing $\phi(z)$ with $[1/(1 + A_2\gamma)]$ (dc_0/dz), and using the relation

$$\sigma_e = \int_R^{kR} \rho v_z(2\pi r)\, dr \tag{12}$$

gives

$$\tau_1 = c_0\sigma_e + H_e c\bar{c} - K_e \frac{dc_0}{dz} \tag{13}$$

where

$$H_e = \frac{\alpha\beta_T\rho g(2\pi R)(\Delta T)^2[R(k - 1)]^3}{6!\,\mu T} F(k) \tag{14}$$

$$K_e = \frac{\beta_T{}^2\rho g^2(2\pi R)(\Delta T)^2[R(k - 1)]^7[1 - (79/24)A_3{}^2]}{9!\,D\mu^2} G(k) + 2\pi\rho D R^2\left\{\frac{1}{1 + A_2\gamma}\left[\frac{k^2 - 1}{2} - \frac{A_2(k^3 - 1)}{3}\right]\right\} \tag{15}$$

$$F(k) = \frac{180}{(k - 1)^3 \ln k}\int_1^k \left\{\left(A_1 + \frac{1}{\ln k}(\xi^2 - 1) - \frac{\xi^2 \ln \xi}{\ln k} - \frac{1}{\ln k}\left[\frac{1}{\ln k}(k^2 - 1 - k^2 \ln k) + A_1(k^2 - 1)\right]\ln \xi\right\}\xi \ln \frac{\xi}{r}\, d\xi \tag{16}$$

$$G(k) = \frac{-22680}{(k-1)^7[1-(79/24)A_3{}^2](1+A_2\gamma)} \int_1^k \left\{\left(A_1 + \frac{1}{\ln k}\right)(\xi^2 - 1) - \frac{\xi^2 \ln \xi}{\ln k} - \frac{1}{\ln k}\left[\frac{1}{\ln k}(k^2 - 1 - k^2 \ln k) + A_1(k^2-1) + \xi^2\right]\ln \xi\right\}$$
$$\times \left\{\left(\frac{\xi^4}{16} - \frac{\xi^2}{4} + \frac{1}{4}\ln\frac{\xi}{\gamma} - \frac{\gamma^4}{16} + \frac{\gamma^2}{4}\right) + A_2\left[\left(\frac{\xi^5}{25} - \frac{\xi^3}{9} + \frac{2}{15}\ln\frac{\xi}{\gamma} - \frac{\gamma^5}{25} + \frac{\gamma^3}{9}\right)\right] - \frac{1}{\ln k}\left[\left(\frac{\xi^4 \ln \xi}{16} - \frac{\xi^4}{32} - \frac{1}{16}\ln\frac{\xi}{\gamma} - \frac{\gamma^4 \ln \gamma}{16} + \frac{\gamma^4}{32}\right)\right.$$
$$\left. + A_2\left(\frac{\xi^5 \ln \xi}{25} - \frac{2\xi^5}{125} + \frac{1}{25}\ln\frac{\xi}{\gamma} - \frac{\gamma^5 \ln\gamma}{25} + \frac{2\gamma^5}{125}\right)\right]$$
$$- \frac{1}{\ln k}\left[\frac{1}{\ln k}(k^2 - 1 - k^2 \ln k) + A_1(k^2 - 1)\right]$$
$$\times \left[\left(\frac{\xi^2 \ln \xi}{4} - \frac{\xi^2}{4} - \frac{1}{4}\ln\frac{\xi}{\gamma} - \frac{\gamma^2 \ln\gamma}{4} + \frac{\gamma^2}{4}\right)\right.$$
$$\left.\left. + A_2\left(\frac{\xi^2 \ln \xi}{9} - \frac{2\xi^3}{27} - \frac{1}{9}\ln\frac{\xi}{\gamma} - \frac{\gamma^3 \ln\gamma}{9} + \frac{2\gamma^3}{27}\right)\right]\right\}\xi \, d\xi \tag{17}$$

$$A_3 = \frac{72\sigma_e \mu}{2\rho\beta_T \pi R^4 (\Delta T) g (k-1)^3} \tag{18}$$

Since $|c(1, z) - c(k, z)|$ is small compared with the concentration difference between the top and bottom ends, we may make the approximation $c_0(\gamma, z) \approx c_b(z)$, in which $c_b(z)$ is defined by

$$c_b(z) = \frac{2\pi R^2 \rho}{\sigma_e} \int_1^k c v_z \xi \, d\xi \tag{19}$$

in the enriching section. By making this approximation, Eq. (13) can be written as

$$\tau_1 = \sigma_e c_b + H_e c\bar{c} - K_e \frac{dc_b}{dz} \tag{20}$$

Since τ_1 and σ_e are constants at the steady-state, and $(\tau_1/\sigma_e) = c_{fe}$ everywhere in the enriching section, Eq. (20) becomes

$$H_e\left[c\bar{c} - \frac{\sigma_e}{H_e}(c_{fe} - c_b)\right] = K_e \frac{dc_b}{dz} \tag{21}$$

The integration of Eq. (21) with $c\bar{c}$ held constant, which satisfies the boundary conditions:

$$\text{at } z = 0, \qquad c_b = c_i \tag{22}$$

$$\text{at } z = z_{fe}, \qquad c_b = c_{fe} \tag{23}$$

is

$$c_{fe} - c_i = c\bar{c}[1 - \exp(-\sigma_e z_{fs}/K_e)]\frac{H_e}{\sigma_e} \tag{24}$$

By making a similar analysis in the stripping section, we obtain

$$c_{fs} - c_i = -c\bar{c}[1 - \exp(-\sigma_s z_{fs}/K_s)]\frac{H_s}{\sigma_s} \tag{25}$$

where

$$H_s = H_e|_{\sigma_e \to -\sigma_s} \tag{26}$$

$$K_s = K_e|_{\sigma_e \to -\sigma_s} \tag{27}$$

Combining Eqs. (24) and (25), we obtain

$$\Delta = c_{fe} - c_{fs} = c\bar{c}\left\{[1 - \exp -(\sigma_e z_{fe}/K_e)]\frac{H_e}{\sigma_e} + [1 - \exp(-\sigma_s z_{fs}/K_s)]\frac{H_s}{\sigma_s}\right\} \tag{28}$$

For the important case in which c is everywhere within the range of 0.3 to 0.7 and in which therefore $c\bar{c}$ is never far from 1/4, Eq. (28) becomes

$$\Delta = [1 - \exp -(\sigma_e z_{fe}/K_e)]\frac{H_e}{4\sigma_e} + [1 - \exp(-\sigma_s z_{fs}/K_s)]\frac{H_s}{4\sigma_s} \tag{29}$$

Equation (29) is the equation of separation of a binary solution in a continuous concentric-tube thermal diffusion column operating under steady-state conditions.

TRANSPORT COEFFICIENTS

Since operation at high flow rates in a continuous thermal diffusion column is very inefficient, moderate flow rates are usually used. Hence, neglecting the flow rate terms in the transport coefficients H_e, H_s, K_e, and K_s should not lead to serious error. In this case, $A_1 = A_1'$, $A_2 = A_3 = 0$, and thus

$$H = H_e = H_s = H_0F'(k) \tag{30}$$

$$K = K_e = K_s = K_0G'(k) \tag{31}$$

where

$$H_0 = \frac{(2\pi R)\alpha\rho\beta_T g(\Delta T)^2[R(k-1)]^3}{6!\,\mu\bar{T}} \tag{32}$$

$$K_0 = \frac{(2\pi R)\rho\beta_T{}^2 g^2(\Delta T)^2[R(k-1)]^7}{9!\,D\mu^2} \tag{33}$$

$$F'(k) = [F(k)]_{A_1 = A_1'} \tag{34}$$

$$G'(k) = [G(k)]_{A_2 = A_3 = 0} \tag{35}$$

$$A_1'(k) = [A_1]_{\sigma_e = \sigma_s = 0} \tag{36}$$

Here we neglect the second term of Eq. (15) representing remixing due to ordinary diffusion in the transport direction as compared with the convective term given in the first term of the same equation.

If, in addition, the feed is introduced at the midpoint of the column, i.e., $z_{f_e} = z_{f_s} = \frac{1}{2}h$, and the flow rates in both the enriching section and the stripping section are kept the same, i.e., $\sigma = \sigma_e = \sigma_s$, then Eq. (29) reduces to

$$\Delta = \frac{H}{2\sigma}\left[1 - \exp\left(-\frac{\sigma h}{2K}\right)\right] \tag{37}$$

The modifying factors $F'(k)$ and $G'(k)$, which are functions of k only, are still in integrating forms. The integration is simple in principle, but is much more complicated in execution. For convenience, the integration was performed numerically by the digital computer and the results are plotted in Fig. 2.

The curvature of concentric-tube column decreases as k does. When k approches unity, the concentric-tube column is nearly a flat-plate one. At $k = 1$, either the annular spacing or the curvature of the concentric-tube column is zero. When the latter occurs, the thermal diffusion column is exactly a flat-plate one, while, when the former happens, the operation of thermal diffusion is meaningless. Therefore there is a singular point at $k = 1$ and the values of $F'(1)$ and $G'(1)$ cannot be obtained from the above calculation. However, it is shown in Fig. 2 that $F'(k)$ and $G'(k)$ increase as k does, and the extensions of $F'(k)$ and $G'(k)$ as shown by the dotted lines

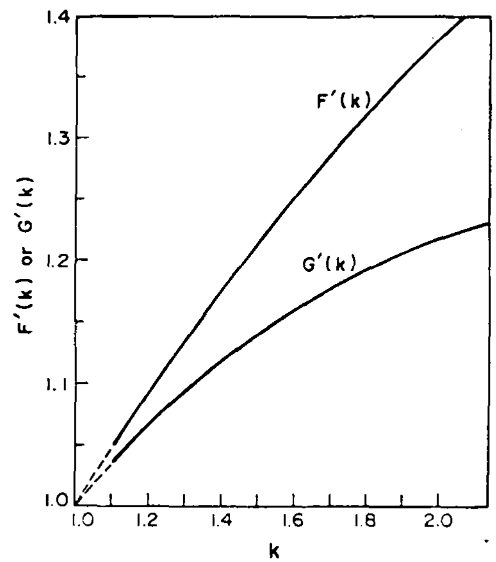

FIG. 2. Graphical representation of modifying factors.

in the direction of decreasing k reach the point (1, 1), i.e., $F'(1) = 1$ and $G'(1) = 1$. Consequently, for a flat-plate column, Eq. (37) reduces to

$$\Delta = \frac{H_0}{2\sigma}\left[1 - \exp\left(-\frac{\sigma h}{2K_0}\right)\right] \tag{38}$$

This is the same result as that obtained by Yeh and Ward (*8*) in which they assumed that the curvature in the concentric-tube column is negligible.

CONCLUSION

On the basis of this study, the following conclusions were reached.

1. A generalized equation of separation, Eq. (28), applicable to the cases $0.3 \leqslant c \leqslant 0.7$, $c\bar{c} \approx 0.25$, has been derived.
2. A modified equation of separation, Eq. (37), applicable only to moderate flow rates, was obtained from Eq. (28). Operation under high flow rates, however, is very inefficient and therefore Eq. (37) is the practical equation of separation for a concentric-tube thermal diffusion column.

3. The solution obtained by Yeh and Ward (8), Eq. (38), is a special case of Eq. (37).

4. For practical calculation in concentric-tube thermal diffusion columns, the results obtained by Yeh and Ward can be used with the transport coefficients multiplied by the modifying factors shown in Fig. 2.

5. The modifying factors are not affected by the flow rate terms in the transport coefficients for moderate flow operations.

SYMBOLS

A_1, A_1', A_2, A_3	system constants, evaluated by Eq. (2), (34), (10), (18)
$c, \bar{c}$	fraction of component 1, 2 in a binary solution
c_b	average concentration, defined by Eq. (19)
c_{fe}, c_{fs}	fraction of component 1 in the product stream exiting from the enriching, stripping section.
c_i	c at $z = 0$
c_0	c at $\xi = \gamma$
D	ordinary diffusion coefficient
$F(k), F'(k)$	system constants, evaluated by Eq. (16), (34)
$G(k), G'(k)$	system constants, evaluated by Eq. (17), (35)
g	gravitational acceleration
H, H_e, H_0, H_s	system constants, evaluated by Eqs. (30), (14), (32), (26)
h	column height
J_r	total flux of component 1 in r-direction.
$J_{r-\mathrm{OD}}$	mass flux of component 1 in r-direction due to ordinary diffusion
$J_{r-\mathrm{TD}}$	mass flux of component 1 in r-direction due to thermal diffusion
$J_{z-\mathrm{OD}}$	mass flux of component 1 in z-direction due to ordinary diffusion
K, K_e, K_0, K_s	system constants, evaluated by Eqs. (31), (15), (33), (27)
kR	inside diameter of outer tube
R	outside diameter of inner tube
r	axis of radial direction
T	absolute temperature
T_1, T_2	temperature of cold, hot wall
$\bar{T}$	reference temperature
v_z	general velocity distribution function

z	axis of transport direction
z_{fe}, z_{fs}	length of the enriching, stripping section

Greek Letters

α	thermal diffusion constant
β_T	$-(\partial\rho/\partial T)$
γ	$(k + 1)/2$
Δ	difference in concentration of top and bottom products
ΔT	$T_2 - T_1$
$\phi(z)$	function defined by $[1/(1 + A_2\gamma)](dc_0/dz)$
ξ	r/R
μ	viscosity
ρ	mass density
σ	mass flow rate
σ_e, σ_s	mass flow rate out the enriching, stripping section
τ_1	mass transport of component 1 along z-direction

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